Exam. Code : 103202
Subject Code : 1028

B.A./B.Sc. 2nd Semester

MATHEMATICS

Paper—I

(Calculus & Differential Equations)

Time Allowed—3 Hours] [Maximum Marks—50

Note :— Attempt **FIVE** questions in all, selecting at least **TWO** questions from each section.

SECTION—A

1. (a) Show that the asymptotes of the curve

$$x^4 - 5x^2y^2 + 4y^4 + x^2 - y^2 + x + y + 1 = 0$$

cut the curve in atmost eight points which lie on a rectangular hyperbola.

(b) Show that the abscissa of the point of inflexion on the curve :

$$x = a - b\cos\theta,\ y = a\theta - b\sin\theta \text{ is } \frac{a^2 - b^2}{a}.$$

5,5

2. (a) Show that at the point (1, –1), there is a cusp on the curve :

$$x^3 + xy^2 + y^3 - 4x^2 + y^2 + 4x + y - 1 = 0$$

(b) In an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ show that the radius of curvature at the end of the major axis is equal to the semi-latus rectum of the ellipse. 5,5

3. (a) Trace the curve $x^3 + y^3 = 3axy,\ a \geq 0$

(b) Evaluate $\int \frac{\sinh x + \cosh x}{\sinh^3 x - \cosh^3 x}\, dx$. 5,5

4. (a) If $I_n = \int (\log x)^n\, dx$, prove that

$$I_n + I_{n-1} = x(\log x)^n.$$

(b) Show that $\int_0^{\pi/2} \sin^{2m}\theta \cos^{2m-1}\theta\, d\theta$

$$= \frac{(2m-2)(2m-4)---4.2}{(4m-1)(4m-3)---(2m+1)},$$ m being a positive integer > 1. 5,5

5. (a) Prove that $\int_0^{\pi} \frac{x\,dx}{a^2\cos^2 x + b^2 \sin^2 x} = \frac{\pi^2}{2ab}$

(b) Find the area above the x-axis and included between the curves $y^2 = 2ax - x^2$ and $y^2 = ax$. 5,5

SECTION—B

6. (a) Find the necessary and sufficient condition that the equation $Mdx + Ndy = 0$ may be exact.

(b) Solve : $y - 2px = f(xp^2)$. 5,5

7. (a) Solve and examine for singular solution of the differential equation :

$(px - y)(x - py) = 2p$.

(b) Find the orthogonal trajectory of the series of parabolas whose equation is $y^2 = 4ax$. 5,5

8. (a) Solve : $(D^3 - 2D^2 + D)y = x^2 \cos x$.

(b) Solve : $(D^2 + a^2)y = \sec ax$, by method of variation of parameters. 5,5

9. (a) Solve in series :

$$(x - x^2)\frac{d^2y}{dx^2} + (1 - 5x)\frac{dy}{dx} - 4y = 0$$

(b) Solve in series :

$$(x + x^2 + x^3)\frac{d^2y}{dx^2} + 3x^2\frac{dy}{dx} - 2y = 0 \qquad 5,5$$

10. (a) Solve in series Bessel's Differential Equation of order n.

(b) Solve : $(x^3D^3 + 3x^2D^2 + xD + 1)y = x \log x$

5,5